Measuring economic welfare

New methods

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CHAPTER 1

An introduction to the money-metric

1.1 Introduction

The basic objective of applied welfare economics is to determine if the introduction of a specific project or economic policy will make an individual or group of consumers better off or worse off than will the available alternatives, including the status quo. The tasks involved are the same, whether we are concerned with evaluating the consequences of some policy that affects the entire economy (e.g., a variation in the level of tariffs on imports) or the effects of a project involving only a small community (e.g., the construction of a road that bypasses a congested urban center). In both instances a procedure is required that will enable the economist to work with observable data about consumer preferences, as revealed in the associated demand functions, in order to draw inferences about how the consumer is affected by changes in relative prices and/or income. The objective of this book is to explain the steps required to utilize such information so as to create an operational welfare measure.

This problem has taxed economists now for over a century. At the microeconomic level of project evaluation, the concept of consumer surplus, based on the area beneath a consumer demand curve, continues to attract widespread interest. At the macroeconomic level, many types of index numbers have been formulated and classified. Yet current thinking hardly yields hopeful conclusions. Many economists deny that it is possible to construct an applied welfare indicator at all. Others have argued that it is possible only in very special and limited circumstances. Still others, perhaps agreeing with that latter body of thought, have claimed that the problems are unimportant, because "reasonable" approximate welfare indicators are available. Subsequent chapters will examine each of these viewpoints in detail. At this point, however, it is sufficient to note that in the past, considerable controversy has surrounded discussions of consumer surplus and index-number techniques. For this reason it is extremely important that it be clear at the outset what we are going to be talking about and what we hope to achieve. The best way to do this is to state five basic criteria that any operational welfare indicator must meet:

1. For an individual or homogeneous group of individuals (i.e., a group possessing identical tastes and expenditure levels) the measure must be capable of ranking all relevant price/quantity situations according to the preferences of the individual or homogeneous group.
2. The measure must take the form of a single metric or scale.
3. The metric or scale must be expressed in monetary units.
4. The welfare indicator must be amenable to calculation in terms of the parameters of ordinary, observable demand functions.
5. Once an indicator meeting the preceding four criteria is constructed, it must be such that it can be aggregated across individuals or homogeneous groups so as to obtain an overall measure of the social desirability of the project or policy under consideration.

Why these criteria? To answer this question, it is necessary to emphasize that the evaluation of any project or policy involves two fundamental steps. First, it is necessary to determine which individuals (or homogeneous groups) appear to gain and which appear to lose as a result of any policy action. Second, given this information about the redistribution of income and consumer satisfaction, it is necessary to decide on the basis of value judgments whether the proposed policy should or should not be undertaken. Criteria 1 through 4 are directed toward taking the first step. Criterion 5 concerns the second step.

Criterion 1 underlies all of modern consumer theory and involves the assumption that all consumers have well-defined tastes. Thus, if we possessed complete information about each individual's preference for one situation over another, we would be able to determine qualitatively whether a project or policy has made that individual worse off or better off. That is, the first criterion establishes the fact that we are interested in the sign of any change in the level of consumer satisfaction. Although many theoretical discussions of economic policy have been confined to this qualitative plane, applied economists require something more. For them, it is necessary to be able to rank all changes on the same scale (criterion 2). This has two effects. First, if we are comparing, say, eight alternative situations, we require eight numbers that will enable us to order the several possibilities. As we shall see, some approaches allow only binary or pairwise comparisons. From an operational point of view, this is extremely cumbersome, because it requires $\sum_{i=2}^{n} (i - 1)$ calculations to be made, where n represents the number of alternatives. In our earlier example, 28 such pairwise comparisons would have to be made. Second, we should be able to determine not only whether or not A and B have gained but also whether A has gained more or less than B. *We require a metric or scale with which to characterize (on a uniform basis) every individu-*

al's set of preferences. As we shall see later in this volume, there are infinitely many possible metrics that could be used. However, from a pragmatic point of view, it is necessary to express the metric in terms that can be easily interpreted by lay economists and the general public. The most obvious approach is to seek a measure that can be expressed in monetary units (criterion 3). This property has considerable pedagogic value and will enable us to make the direct, quantitative comparisons that we seek to undertake. In addition, it represents a significant step toward the achievement of criterion 5. If we are interested in calculating social welfare functions, it is a straightforward matter to attach weights (however complicated) to the money-metric indicators associated with each individual or homogeneous group so as to obtain an overall aggregate index. The weights, of course, become an explicit representation of value judgments and enable one to compare the policy recommendations that will arise when alternative weighting schemes (i.e., alternative value judgments) are assumed. This topic will be discussed in Chapter 8.

Now let us turn to the fourth criterion. The theory of consumer behavior tells us that there is a one-to-one correspondence between the ordinal structure of consumer preferences and the structure of consumer demand functions. This result obviously must play a central role in the construction of an operational welfare indicator. It means that there is, in principle, a link between the observable data contained in the demand functions and economic welfare. Unfortunately, by itself, this fact turns out to be of little assistance in applied welfare economics. Until now, economists have not been able to devise a straightforward, manageable procedure for expressing the link between the demand functions and preferences in terms of *elementary* mathematical functions. An elementary function is one that can be constructed from "polynomials, exponentials, logarithms, trigonometric or inverse trigonometric functions in a finite number of steps by using the operations of addition, subtraction, multiplication, division or composition" (Apostol, 1967). Such elementary relationships are fundamental if we are to systematically characterize available price and expenditure data by means of modern econometric techniques. The use of consumer surplus represents a heroic attempt to achieve this objective. Unfortunately, it suffers from two serious defects: (a) Its logical underpinnings are extremely weak. (b) Its performance, in practice, is bound to be subject to substantial error. These points will be discussed in detail in subsequent chapters.

Faced with this situation, we may be tempted to conclude that we should not concern ourselves with this particular criterion and, instead, should simply follow the methodology of the theorist and base any evaluation procedures on the assumption that we know the ordinal preference function describing consumer behavior. However, this approach is not entirely satisfactory either. As we shall determine in the next chapter, there are many circumstances in which it is not possible to express the money-metric in terms of elementary mathematical functions, even though we may possess a function that exactly characterizes consumer preferences. Thus, the problem facing those who desire to start from demand functions reemerges, albeit in a slightly different guise. However, the solution is the same. As we shall see in Chapter 3, it is possible to derive a money-metric in terms of the parameters of ordinary demand functions by using some very basic numerical procedures. The elusive task of trying to express a money measure of welfare in terms of simple algebraic formulas will therefore be abandoned in favor of more complex but more fruitful techniques.

1.2 Some basic assumptions

Considerable effort has been expended by economists in formalizing the properties that characterize consumer preferences and, in particular, in identifying very general conditions under which they can exist. The analysis contained in this book will fall short of the most general treatment, however, because it is not possible to subject it to the techniques of applied econometric demand analysis. Preferences exhibiting satiation or kinks will be assumed away, as will lexicographic orderings. To be explicit, we shall assume that the problems with which we deal in subsequent chapters exhibit the following properties:

Property 1. The services generated in the process of consuming commodities can be expressed as nonnegative real numbers. This condition will also be placed on commodity prices and total expenditure. Thus, certain commodities or services that have a zero nominal price are ruled out, as are those commodities for which no consumption takes place. Given the level of aggregation that characterizes even the best available data, it is not likely that this assumption could be avoided, even if that were believed to be crucial (which it is not) to the analysis of this volume.

Let X_i represent the quantity consumed for commodity i. Then the approach to be adopted assumes that each individual consumer possesses a preference function

$$U = U(X_1, \ldots, X_n) \tag{1.1}$$

which possesses the following properties:
Property 2. The consumer is capable of ordering all possible combinations or market baskets of commodities. Thus, for every possible pair of such baskets, denoted as Z_i and Z_j, one of the following relationships must hold:

1. Z_i is preferred to Z_j.
2. Z_j is preferred to Z_i.
3. The consumer is indifferent between Z_i and Z_j.

Property 3. For all possible market baskets, if Z_i is preferred to Z_j and Z_j is preferred to Z_k, then Z_i is preferred to Z_k. This has the effect of eliminating the possibility that the three situations listed under Property 2 could exist simultaneously.
Property 4. The preference function U that orders the various market baskets is continuous. That is, no gaps exist in the ordering.
Property 5. The preference function U is increasing over the various commodities. More is preferred to less.
Property 6. The *marginal rate of substitution* indicates the rate at which one commodity substitutes for another along any indifference surface, if the quantities of all other commodities are held constant. Thus, for any pair of commodities, i and j, we shall assume that the marginal rate of substitution of i for j decreases as X_j increases. The interpretation is that as the ratio of X_i to X_j decreases, the consumer becomes more reluctant to give up X_i so as to maintain his level of satisfaction constant. That is, the consumer is willing to give up fewer and fewer units of X_i to obtain one additional unit of X_j.

Finally, we need to make two assumptions that will be of crucial significance for the computational procedures to be discussed in Chapter 3. First, we shall assume that consumer preferences can be represented by a function that is analytic over the region of price and income variation. That is, we shall assume that any preference function to be considered possesses derivatives of all orders for all values of prices and total expenditure to be considered. However, the main characteristic of an analytic function is that it may be represented by a Taylor series expansion that is covergent as its order approaches infinity. A Taylor series of infinite order is of little operational significance until it is truncated. Because there is always the possibility that a series of order k will neglect the impact of very large derivatives of order greater than k, we shall restrict the functions to be studied by making one further assumption: The absolute value of the remainder term associated with any kth-order Taylor series expansion monotonically tends to zero as k approaches infinity.

Although these two assumptions do restrict the domain of mathematical functions to be considered, it is not unreasonable to conjecture that human behavior does tend to satisfy the conditions noted. Both Kannai (1974) and Mas-Colell (1974) have examined the mathematical properties of such functions and have concluded that they are not unreasonable. With respect to the assumption of monotonic convergence of the Taylor series, it is highly unlikely that human behavior will be heavily dominated by a high-order derivative of a consumer preference function with respect to its arguments. Indeed, for realistic values of prices and income, the various preference functions underlying econometric estimation of demand systems all possess the properties stated in the following composite assumption:

Property 7. Consumer preferences can be expressed as an analytic function. Further, the absolute value of the remainder term of any Taylor series monotonically tends toward zero as the order of the series tends toward infinity.

Thus, the framework that we shall adopt is not completely general. Nevertheless, these assumptions provide the basis for contemporary work in the field of applied demand analysis and for most discussions of consumer surplus or index-number techniques. Equally important from the point of view of this volume is the fact that these assumptions enable us to focus very clearly on the basic problem: It is one thing to be able to draw an indifference map such as that in Figure 1.1; it is quite another thing to be able to express it in terms of elementary functions.

1.3 The money-metric

Although the search for a monetary representation of consumer preferences has generated considerable controversy over the past 100 years or so, such a measure is really quite simple to visualize. Indeed, the concept is already widely known, but, unfortunately, widely neglected and/or misinterpreted. Consider Figure 1.1, which depicts (a) several representative indifference surfaces satisfying the assumptions previously discussed, (b) budget lines (on the assumption of constant prices), and (c) the associated income expansion path. Suppose also that the incomes represented in the diagram are those shown in Table 1.1. It is fairly easy to appreciate that we may adopt any numerical ordering pattern for the indifference surfaces provided that the chosen representation is always monotonic and increasing. That is, suppose $\phi = \phi(X_1, \ldots, X_n)$ is some function describing the indifference surfaces in Figure 1.1. Then any transformation

Table 1.1 *Transformation*

Indifference surface	Expenditure level (constant prices)	R	S	T
I	100	100	100	100
II	200	170	220	200
III	300	220	370	300
IV	400	250	550	400

$U = U[\phi(X_1, \ldots, X_n)]$ will also do, provided that $\partial U/\partial\phi$ is greater than zero. The points of tangency of the indifference surfaces with the budget lines remain unchanged irrespective of the transformation chosen. Consumer behavior remains unaffected. This property is extremely important. In the past, many economists thought that satisfaction could be quantified in terms of measured units of pain or pleasure. However, the fact that we can represent a given preference system by any arbitrary increasing monotonic transformation implies that a cardinal representation of consumer satisfaction is not necessary. An ordinal approach is all that is required. Although this is true, it does not necessarily exclude the possibility that some cardinal representation may be of interest in its own right. To appreciate that this is so, again consider Table 1.1, which shows three alternative numbering schemes R, S, and T that might be used to characterize the indifference surfaces shown in Figure 1.1.

Under transformation R we note that although the utility indicator chosen increases as money expenditure increases, it does so at a decreasing rate. That is, the marginal utility of money decreases as total expenditure increases. Under transformation S, the utility indicator increases at an increasing rate. The marginal utility of money increases as total expenditure increases. However, under transformation T, the marginal utility of money remains constant as expenditure increases. The latter representation immediately suggests a money-metric interpretation. Suppose that the slopes of the budget lines indicate the prices that hold in some initial situation, say that labeled 1 in Figure 1.1. Then it is possible to order all alternatives in the following way. Suppose that some policy change enables the price/quantity situation at point 3 to be achieved. Then we can say that this new situation generates a level of satisfaction that is equivalent to a move from point 1 to point 2 brought about solely by a change in money income. The same can also be said for point 5. It is immediately obvious that all price/quantity situations can be ranked in this way,

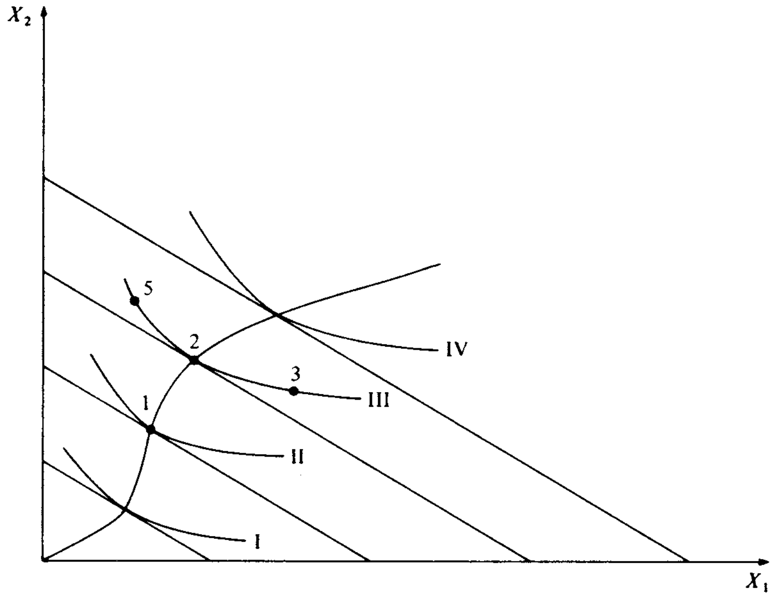

Figure 1.1

because it is always possible to find some level of expenditure, given initial prices, that generates the same level of satisfaction as the policy change. This scale is identical with the transformation T shown in Table 1.1. The metric used to characterize consumer preferences is the level of expenditure required to achieve any level of satisfaction given initial prices. The difference between this level of expenditure and the initial level is called the *equivalent variation.* Thus, a move from situation 1 to situation 3 is equivalent to an income gain of 100 dollars. By calculating this measure for all situations of interest, the alternatives can be ranked not just ordinally but in terms of a monetary unit (i.e., in terms of a metric) that is easily understood.

The advantage of this particular formulation is that it allows the policymaker or project analyst to compare the monetary gains or losses accruing to consumers with the monetary cost of financing that project or policy. In other words, the approach to be developed in this volume is capable of forming a meaningful basis for what is conventionally called cost–benefit analysis. For example, consider the situation illustrated in Figure 1.2. Let us suppose that a project is contemplated that will lead to an increase in the consumption of X_1 and X_2. Suppose also that the funds required to undertake this project

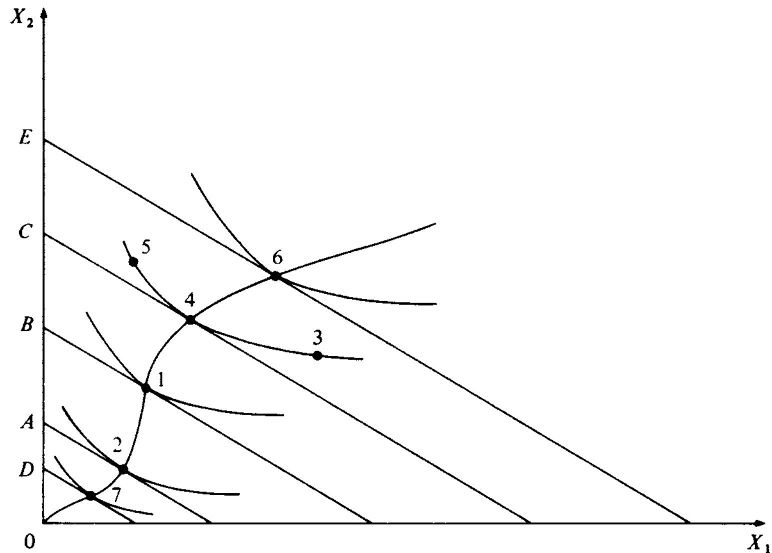

Figure 1.2

are raised by a lump-sum income tax. Given that the initial situation is depicted as point 1 in Figure 1.2, the imposition of the tax, *ceteris paribus,* lowers welfare to point 2. If we normalize the price of X_2, so that its initial value is one, then the value of the tax is calculated by the distance AB. The funds having been raised, the project is now introduced, with the result that the economy moves from point 2 to point 3. This is equivalent to an increase in expenditure equal to AC such that the economy moves from point 2 to point 4. In other words, at a cost of AB, the economy enjoys gains equivalent to AC, thereby producing net gains of BC.

If several projects are under consideration, then similar calculations of costs and benefits can be made for each, and the results can be compared so as to reveal which is best. Consider, for example, one alternative (a move from 1 to 7) that costs an amount equal to BD and generates gains equivalent to DE (a move from 7 to 6). The net gain is thus BE, which is greater than the BC associated with the project discussed earlier.

To many readers, this result will appear quite surprising. Yet the concept of the equivalent variation is well known in the literature dealing with various aspects of welfare economics. It was first defined as such by Hicks as part of a discussion that took place in the 1940s as economists attempted to clarify his celebrated rehabilitation of consumer surplus. The major analytical issues involved will be examined in Chapters 4 and 5. In 1949, Allen used this measure as the basis for an exact quantity index. Its properties were pointed out in a well-known study by Hurwicz and Uzawa (1971) on the important problem of the integrability of demand functions. Yet, it is only in recent years, following Samuelson's labeling of the measure as the "money-metric" and the attempts by McKenzie and Pearce (1976) to characterize it in terms of the parameters of demand functions, that greater interest has been shown in this concept.

However, if a full understanding of the issues involved is to be achieved, numerous misconceptions must be clarified. Despite widespread awareness of the equivalent variation, economists have not fully appreciated the relationship of this measure to consumer preferences. For example, Debreu (1951, p. 273) wrote that a basic problem in applied welfare economics "comes from the fact that no meaningful metrics exists in the satisfaction space." Boadway (1974), summarizing the conclusions of Silberberg (1972), Mohring (1971), and Burns (1973), concluded that "there can be no measure of welfare change in monetary units which is independent of the path of integration." This is a very strong result that in subsequent chapters will be shown to be mathematically incorrect. That it is intuitively incorrect can be appreciated from the diagrammatic analysis of the equivalent variation previously discussed. Yet, having determined that a money measure exists, another misconception must be cleared away. As we shall determine in the next chapter, Willig's claim (1976, p. 589) that the equivalent variation is "unobservable" is clearly wrong.

Much discussion has also been confused by the preference of most cost–benefit analysts and index–number theorists for the conceptually identical notions of "willingness to pay" and the *compensating variation.* This indicates the amount of money that a consumer will be willing to pay, following some policy change, so as to return to the initial level of satisfaction. It is important that the compensating variation and equivalent variation not be confused. For one thing, the reference points for the two are entirely different. For the equivalent variation, an arbitrarily chosen vector of prices serves to define the base situation, whereas with the compensating variation it is an arbitrarily chosen base level of satisfaction. The latter will be consistent with a large number of alternative price vectors. More important, however, is the fact that, unlike the equivalent variation, the compensating variation cannot be used to construct a metric that meets the first criterion set out earlier in this chapter. The compensating variation is not an ordinal welfare metric. It is true, as several writers have pointed out, that it could conceivably be used for binary comparisons. However, this involves an extremely cumbersome procedure. As we have already noted, if there are n alternatives, $\sum_{i=2}^{n}(i-1)$ pairwise calculations will have to be made if the compensating variation is to be used. The results will have to be presented in matrix form. In contrast, if the equivalent variation is used, only n calculations will be required, and the results can be presented in vector notation in the same way that index numbers are presented in statistical abstracts. In other words, the equivalent variation is a money-metric, whereas the compensating variation is not.

1.4 Summary

Two objectives have been achieved in this chapter. First, we have listed a set of assumptions characterizing consumer behavior. Second, we have shown that, in principle, consumer preferences can be characterized by a money-metric and that this is related to the well-known equivalent variation.